

TECHNICAL NOTES

Heat transfer in forced oscillatory compressible flow past a yawed cylinder

G. N. SARMA and REETA SRIVASTAVA

Department of Mathematics, University of Roorkee, Roorkee, India

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INTRODUCTION

IN THE present age of rockets and artificial satellites, the study of compressibility and energy dissipation in aerodynamic, oscillatory, thermal boundary layers is of great importance. Such studies of periodic incompressible and compressible flows have been made by many authors such as Lighthill [1], Rott and Rosenweig [2], Ackerberg and Phillips [3] and Sarma [4-6]. The present work aims to find solutions of three-dimensional, oscillatory, compressible boundary-layer flow past a yawed cylinder of infinite span when the temperature of the cylinder is oscillatory about a steady mean. Compressible boundary layers being known for their mathematical complexity, the analysis is done in terms of the Stewartson's variables [7]. The equations are linearised as in Lighthill [1] and series solutions have been obtained in powers of Mach number as in Hawarth [9]. The non-uniformities and inhomogeneities present in the problem force us to find asymptotic solutions for small and large values of the frequency parameter ξ . But no attempt has been made here to model the complicated eigen solutions similar to those of Brown and Stewartson [8] to smooth the joining of the solutions for small and large frequencies.

The analytical and numerical analysis given in this paper help us to study the heat transfer effects due to different shapes of the cross-section of the cylinder, due to frictional heating and due to other parameters present in the problem. The ordinary differential equations occurring here are solved numerically by the shooting method of Reshotko and Beckwith [9] and Cohen and Reshotko [10]. Important numerical values are tabulated, omitting lengthy equations and expressions. The behaviour of heat transfer and skin friction coefficients are represented graphically for different values of the parameters $\beta, \sigma, m_1, M, W/V, \gamma, T_w/T_s$.

EQUATIONS AND STEADY-STATE SOLUTION

The boundary-layer equations given in ref. [6] governing the unsteady, three-dimensional, compressible laminar flow past a yawed infinite cylinder, in Stewartson's variables, are

$$\frac{a_\infty}{a_s} \frac{\partial^2 \psi}{\partial X \partial t} + \frac{a_\infty}{a_s^2} \frac{\partial a_\infty}{\partial X} \left(\frac{\partial \psi}{\partial Y} \right)^2 \frac{\partial X}{\partial x} + \frac{a_\infty^2}{a_s^2} \frac{\partial X}{\partial x} \times \left[\frac{\partial \psi}{\partial Y} \frac{\partial^2 \psi}{\partial X \partial Y} - \frac{\partial \psi}{\partial X} \frac{\partial^2 \psi}{\partial Y^2} - \frac{\partial^3 \psi}{\partial Y^3} \right] = \frac{TU}{T_\infty} \frac{dU}{dX} \frac{\partial X}{\partial x} \quad (1)$$

$$\frac{\partial w}{\partial t} + \frac{a_\infty}{a_s} \frac{\partial X}{\partial x} \left[\frac{\partial \psi}{\partial Y} \frac{\partial w}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial w}{\partial Y} - \frac{\partial^2 w}{\partial Y^2} \right] = 0 \quad (2)$$

$$\frac{\partial T}{\partial t} + \frac{a_\infty}{a_s} \frac{\partial X}{\partial x} \left[\frac{\partial \psi}{\partial Y} \frac{\partial T}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial T}{\partial Y} \right] + \frac{\partial \psi}{\partial Y} \frac{dU}{dX} \frac{\partial X}{\partial x} \left(\frac{UTa_\infty}{C_p T_\infty a_s} \right) = \frac{a_\infty}{a_s} \frac{\partial X}{\partial x} \left[\frac{1}{\sigma} \frac{\partial^2 T}{\partial Y^2} + \frac{1}{C_p} \frac{a_\infty^2}{a_s^2} \left(\frac{\partial^2 \psi}{\partial Y^2} \right)^2 + \frac{1}{C_p} \left(\frac{\partial w}{\partial Y} \right)^2 \right] \quad (3)$$

where

$$X = \int_0^x \left(\frac{a_\infty}{a_s} \right)^{(3\gamma-1)/(\gamma-1)} dx, \quad Y = \frac{a_\infty}{a_s \sqrt{v_s}} \int_0^y \frac{\rho}{\rho_s} dy$$

$$\frac{\partial \psi}{\partial Y} = \frac{a_s u}{a_\infty}, \quad T_\infty + \frac{U^2}{2C_p} = T_s \text{ (a constant)}$$

$$T_\infty = T_s \left(1 + \frac{\gamma-1}{2} M^2 \right) + O(M^4)$$

$$U = V \left(1 - \frac{\gamma-1}{4} M^2 \right) + O(M^4).$$

To solve these complicated equations, just as in Lighthill [1] and Howarth [11], the periodic compressible flow is assumed to be a perturbed, steady, incompressible flow. So we assume the following boundary conditions and structures for the solutions

$$\Psi = \frac{\partial \Psi}{\partial Y} = w = 0,$$

$$T = T_w + \varepsilon T_s \delta X^m e^{i\omega t} \quad \text{at} \quad Y = 0$$

$$\frac{\partial \Psi}{\partial Y} \rightarrow V(X) = \alpha_0 X^{m_0},$$

$$T \rightarrow T_\infty(X), \quad w \rightarrow W \text{ (a constant) as } Y \rightarrow \infty$$

NOMENCLATURE

a	sound velocity
C_p	specific heat
i	$\sqrt{-1}$
m_0, m_1	constants
M	Mach number
t	time
T	temperature
u, w	chord and spanwise velocities
V	$a_s M$
x, y	coordinates along and perpendicular to the chord.

Greek symbols	
ρ	density
ω	frequency
γ	ratio of specific heats
ν	kinematic viscosity
μ	coefficient of viscosity
σ	Prandtl number
α_0, α_1	constants.
Subscripts	
s	point in main stream
∞	outside the layer
W	at the wall.

$$\Psi = \alpha A_0 - \frac{\alpha}{2} \left(A_1 + \frac{W^2}{V^2} A_2 \right) \left(\frac{\gamma-1}{2} \right) M^2 + \varepsilon e^{i\omega t} \left[F_0 + \left(F_1 + \frac{W^2}{V^2} F_2 \right) \left(\frac{\gamma-1}{2} \right) M^2 \right]$$

$$T = T_s B_0 - T_s \left(B_1 + \frac{W^2}{V^2} B_2 \right) \left(\frac{\gamma-1}{2} \right) M^2 + \varepsilon e^{i\omega t} \left[G_0 + \left(G_1 + \frac{W^2}{V^2} G_2 \right) \left(\frac{\gamma-1}{2} \right) M^2 \right]$$

$$w = WC_0 + W \left(C_1 + \frac{W^2}{V^2} C_2 \right) \left(\frac{\gamma-1}{2} \right) M^2 + \varepsilon e^{i\omega t} \left[H_0 + \left(H_1 + \frac{W^2}{V^2} H_2 \right) \left(\frac{\gamma-1}{2} \right) M^2 \right]$$

when

$$|\varepsilon| < 1, \quad \alpha = [2\alpha_0 X^{m_0+1}/(m_0+1)]^{1/2}$$

and F_i, G_i, H_i are functions of (X, Y, ω) and satisfy a system of differential equations determined by the equations of motion (1)–(3). The terms containing W^2/V^2 represent the part due to frictional heating (dissipation), the terms containing M^2 represent the part due to compressibility, the terms containing ε represent the part due to periodic flow and terms independent of ε represent the part due to steady flow. The functions A_i, B_i, C_i are functions of

$$\eta = \left[\frac{\alpha_0}{2} (m_0+1) X^{m_0-1} \right]^{1/2} Y$$

associated with the steady state and which satisfy the following equations

$$A_0'' + A_0 A_0' + \beta(B_0 - A_0') = 0, \quad A_0(0) = A_0'(\infty) = 0, \quad A_0'(\infty) = 1$$

$$B_0' + \sigma A_0 B_0' = 0, \quad B_0(0) = T_w/T_s, \quad B_0(\infty) = 1, \quad \beta = \frac{2m_0}{m_0+1}$$

$$C_0' + A_0 C_0' = 0, \quad C_0(0) = 0, \quad C_0(\infty) = 1$$

$$A_1'' + A_0 A_1'' - 4\beta A_0' A_1' + \left(\frac{5m_0+1}{m_0+1} \right) A_0' A_1 + 2 \left(\frac{2\gamma-1}{\gamma-1} \right) (A_0 A_0'' + A_0''') + \beta \left[\frac{1}{2} \left(B_1 + \frac{2\gamma-1}{\gamma-1} B_0 \right) - 2 \left(\frac{4\gamma-3}{\gamma-1} \right) A_0'^2 \right] = 0$$

$$B_1' - \sigma \left[2\beta(B_1 - B_0) A_0' - A_0 B_1' - \frac{2\gamma-1}{\gamma-1} \left(A_0 B_0' + \frac{1}{\sigma} B_0'' \right) - \frac{1}{2} \left(\frac{5m_0+1}{m_0+1} \right) B_0' A_1 \right] = 0$$

$$C_1' + A_0 C_1' - 2\beta A_0' C_1 - \frac{2\gamma-1}{\gamma-1} (A_0 C_0' + C_0'') - \frac{5m_0+1}{2(m_0+1)} C_0' A_1 = 0$$

$$A_2'' + A_0 A_2'' - 4\beta A_0' A_2' + \frac{5m_0+1}{(m_0+1)} A_0' A_2 + \frac{\beta}{2} B_2 = 0$$

$$B_2' + \sigma \left[A_0 B_2' - 2\beta A_0 B_2 + \frac{5m_0+1}{2(m_0+1)} B_0 A_2 - 2C_0'' \right] = 0$$

$$C_2' + A_0 C_2' - 2\beta A_0' C_2 - \frac{5m_0+1}{2(m_0+1)} C_0' A_2 = 0$$

with

$$A_n(0) = A_n'(0) = B_n(0) = C_n(0) = C_n(\infty) \quad \text{for } n = 1, 2$$

$$A_1'(\infty) = B_1(\infty) = 1, \quad A_2'(\infty) = B_2(\infty) = 0.$$

These equations are solved numerically by the shooting method [9, 10]. Table 1 gives some of the values.

ANALYSIS OF OSCILLATORY PART

To study the oscillatory part of motion, as in Ackerberg and Phillips [3], we find the following two dissimilar asymptotic solutions for F_i, G_i, H_i , one for small $\xi (= \omega X/V)$ and the other for large ξ .

$$(F_0, F_1, F_2) = \alpha \alpha_1 \xi^{m_1} \sum_{n=0}^{\infty} (2i\xi)^n (p_n, p_{1,n}, p_{2,n}) \quad \text{as } \xi \rightarrow 0$$

$$= \alpha \alpha_1 \xi^{m_1} \sqrt{\frac{m_0+1}{2}} \sum_{n=0}^{\infty} \left(\frac{f_n}{\xi^{(n+3)/2}}, \frac{f_{1,n}}{\xi^{(n+1)/2}}, \frac{f_{2,n}}{\xi^{(n+1)/2}} \right) \quad \text{as } \xi \rightarrow \infty$$

$$(G_0, G_1, G_2) = \alpha_1 T_s \xi^{m_1} \sum_{n=0}^{\infty} (2i\xi)^n (q_n, q_{1,n}, q_{2,n}) \quad \text{as } \xi \rightarrow 0$$

$$= \alpha_1 T_s \xi^{m_1} \sum_{n=0}^{\infty} \left(\frac{g_n}{\xi^{n/2}}, \frac{g_{1,n}}{\xi^{n/2}}, \frac{g_{2,n}}{\xi^{n/2}} \right) \quad \text{as } \xi \rightarrow \infty$$

$$(H_0, H_1, H_2) = \alpha_1 W \xi^{m_1} \sum_{n=0}^{\infty} (2i\xi)^n (r_n, r_{1,n}, r_{2,n}) \quad \text{as } \xi \rightarrow 0$$

$$= \alpha_1 W \xi^{m_1} \sqrt{\frac{m_0+1}{2}} \sum_{n=0}^{\infty} \left(\frac{h_n}{\xi^{(n+3)/2}}, \frac{h_{1,n}}{\xi^{(n+1)/2}}, \frac{h_{2,n}}{\xi^{(n+1)/2}} \right) \quad \text{as } \xi \rightarrow \infty$$

where $m_1 = [m/(1-m_0)]$, $\alpha_1 = (\alpha_0/\omega)^{m_1} \delta$; p, q and r are all functions of η and f, g and h are all functions of $Y (= \sqrt{\omega} Y)$ whose differential equations can be obtained from (1)–(3) and satisfy the boundary conditions

$$q_0(0) = g_0(0) = 1, \quad q_n(0) = g_n(0) = 0, \quad \text{for } n > 0,$$

$$\begin{bmatrix} p_n & p_n' & p_{1,n} & p_{1,n}' \\ f_n & f_n' & f_{1,n} & f_{1,n}' \\ 0 & 0 & q_{1,n} & q_{1,n}' \\ r_n & h_n & r_{1,n} & h_{1,n}' \end{bmatrix} \begin{matrix} \eta=0 \\ Y=0 \\ l=1,2 \\ n \geq 0 \end{matrix} = 0,$$

Table 1.

β	$T_w/T_s = 0.2, \sigma = 1$			$T_w/T_s = 2, \sigma = 1$		
	A_0'	$B_0'(0)$	$C_0'(0)$	$A_0'(0)$	$B_0'(0)$	$C_0'(0)$
0	0.46960	0.38476	0.49569	0.46960	-0.48099	0.49569
0.5	0.64762	0.41326	0.53705	1.24323	-0.5909	0.62590
1.0	0.76265	0.42753	0.55789	1.75138	-0.63668	0.68259

$$\begin{bmatrix} p'_n & p'_{1,n} & f'_n & f'_{1,n} \\ q_n & q_{1,n} & g_n & g_{1,n} \\ r_n & r_{1,n} & h_n & h_{1,n} \end{bmatrix}_{\substack{y=0 \\ y=\infty \\ l=1,2 \\ n \geq 0}} = 0.$$

The heat transfer and skin friction coefficients of the oscillatory part which are important to study the cooling and heating effects in the boundary layer are given by

$$T_{1Y}^{(D)} = \frac{T_{1Y} e^{-i\omega t}}{T_s \alpha_1 \sqrt{\omega}}, \quad \left(\frac{\partial T}{\partial Y} \right)_{Y=0} = T_{0Y} + \varepsilon T_{1Y}$$

$$\tau_{1u}^{(D)} = \frac{\tau_{1u} e^{-i\omega t}}{\alpha_1 V \mu \sqrt{\frac{V(m_0+1)}{2v_s X}}}, \quad \left(\mu \frac{\partial u}{\partial y} \right)_{y=0} = \tau_{0u} + \varepsilon \tau_{1u}$$

$$\tau_{1w}^{(D)} = \frac{\tau_{1w} e^{-i\omega t}}{\alpha_1 W \mu \sqrt{\frac{V(m_0+1)}{2v_s X}}}, \quad \left(\mu \frac{\partial w}{\partial y} \right)_{y=0} = \tau_{0w} + \varepsilon \tau_{1w}$$

where (up to the orders of ξ^{m_1+3}, ξ^{m_1-4})

$$T_{1Y}^{(D)} = \sum_{n=0}^3 (2i)^n \xi^{m_1+n} \left\{ q'_n(0) + \left[q'_{1,n}(0) + \frac{W^2}{V^2} q'_{2,n}(0) \right] \times \left(\frac{\gamma-1}{2} \right) M^2 \right\} \text{ as } \xi \rightarrow 0 \quad (4)$$

$$= \sum_{n=0}^5 \xi^{m_1-n/2} \left\{ g'_n(0) + \left[g'_{1,n}(0) + \frac{W^2}{V^2} g'_{2,n}(0) \right] \times \left(\frac{\gamma-1}{2} \right) M^2 \right\} \text{ as } \xi \rightarrow \infty \quad (5)$$

$$\begin{aligned} \tau_{1u}^{(D)} &= \sum_{n=0}^3 (2i)^n \xi^{m_1+n} \left\{ p'_n(0) + \left[p'_{1,n}(0) + \frac{W^2}{V^2} p'_{2,n}(0) \right] \right. \\ &\quad \left. - \frac{2\gamma-1}{\gamma-1} p'_n(0) \right\} \left(\frac{\gamma-1}{2} \right) M^2 \text{ as } \xi \rightarrow 0 \\ &= \sqrt{\frac{m_0+1}{2}} \left(1 - \frac{2\gamma-1}{2} M^2 \right) \sum_{n=0}^5 \xi^{m_1-(n+3)/2} f'_n(0) \end{aligned}$$

$$\begin{aligned} &+ \sqrt{\frac{m_0+1}{2}} \left(\frac{\gamma-1}{2} \right) M^2 \sum_{n=0}^5 \xi^{m_1-(n+1)/2} \\ &\times \left[f'_{1,n}(0) + \frac{W^2}{V^2} f'_{2,n}(0) \right] \text{ as } \xi \rightarrow \infty \quad (6) \end{aligned}$$

$$\begin{aligned} \tau_{1w}^{(D)} &= \sum_{n=0}^3 (2i)^n \xi^{m_1+n} \left\{ r'_n(0) + \left[r'_{1,n}(0) + \frac{W^2}{V^2} r'_{2,n}(0) \right] \right. \\ &\quad \left. - \frac{3\gamma-1}{2(\gamma-1)} r'_n(0) \right\} \left(\frac{\gamma-1}{2} \right) M^2 \text{ as } \xi \rightarrow 0 \\ &= \sqrt{\frac{m_0+1}{2}} \left(1 - \frac{2\gamma-1}{2} M^2 \right) \sum_{n=0}^5 \xi^{m_1-(n+3)/2} h'_n(0) \\ &+ \sqrt{\frac{m_0+1}{2}} \left(\frac{\gamma-1}{2} \right) M^2 \sum_{n=0}^5 \xi^{m_1-(n+1)/2} \\ &\times \left[h'_{1,n}(0) + \frac{W^2}{V^2} h'_{2,n}(0) \right] \text{ as } \xi \rightarrow \infty. \quad (7) \end{aligned}$$

The equations for p , etc. associated with small frequencies are solved numerically by the shooting method [9, 10] and the important numerical values are tabulated in the Appendix for particular sets of values of the parameters occurring. The equations for f , etc. associated with large frequencies are solved by elementary analytical methods. But these are the inner solutions and do not satisfy the boundary conditions at infinity. A formal outer solution and a composite solution can also be obtained just as in Ackerberg and Phillips [3]. The heat transfer and skin friction coefficients calculated from the inner solution will be the same as those calculated from the composite solution itself as $Y \rightarrow 0$.

PARTICULAR PROBLEMS AND CONCLUSIONS

With the help of the above analysis and using the numerical values and the solutions given, the behaviours of $|T_{1Y}^{(D)}|$ and $|\tau_{1u}^{(D)}|$, the magnitudes of heat transfer and skin friction coefficients, are represented graphically in Figs. 1 and 2 for

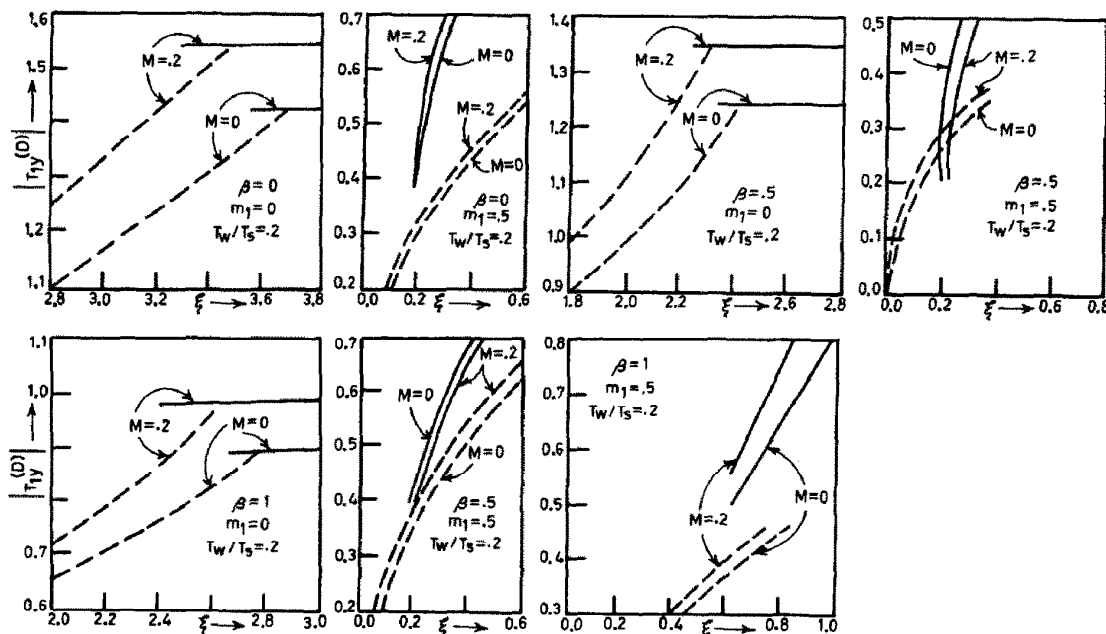


FIG. 1. Heat transfer variations for small and large frequency parameter ξ . The curves ---- are from (4) and the curves — are from (5) when $\sigma = 1, W^2/V^2 = 0.1, \gamma = 1.4$.

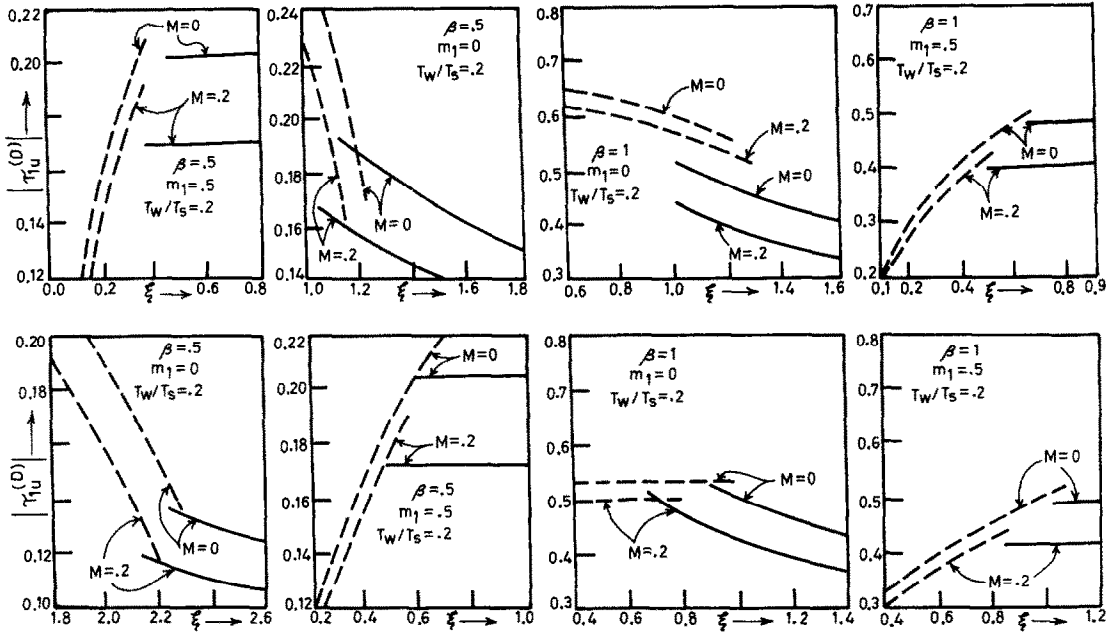


FIG. 2. Skin friction variations for small and large frequency parameter ξ . The curves ---- are from (6) and the curves — are from (7) when $\sigma = 1$, $W^2/V^2 = 0.1$, $\gamma = 1.4$.

small and large values of ξ and for the different parameters on which they depend: (1) the shape of the body β ; (2) the Prandtl number σ ; (3) the perturbation parameter m_1 ; (4) the effects of compressibility M ; (5) the orientation of the cylinder W/V (angle of yaw); (6) the ratio of specific heats γ ; (7) the ratio of the temperatures T_w/T_s . The curves represent the behaviours in three distinct problems of yawed flat plate ($\beta = 0$), yawed wedge ($\beta = 0.5$) and yawed flat plate with a stagnation point ($\beta = 1$).

The curves in Fig. 1 for small and large ξ increase with ξ showing that the heat transfer increases for all values of ξ whereas in Fig. 2, the behaviour of the skin friction coefficient in the x -direction for small and large ξ shows the increasing or decreasing character. But all curves for small ξ show the tendency to join with those for large ξ , although the joining is not smooth enough. To make it smooth, one may use the complicated eigensolutions as suggested by Brown and Stewartson [8].

The compressibility effects, from Fig. 1, in general, increase the heat transfer for both small and large values of ξ . In particular the character of increasing or decreasing with M for small ξ is opposite to that for large ξ . This is evident from graphs for $(\beta, m_1, T_w/T_s) = (0.5, 0.5, 0.2)$, $(0.5, 0.5, 2.0)$, $(1, 0.5, 2)$. Heat transfer coefficient, in general, decreases with the parameter m_1 .

In all the particular cases illustrated in Fig. 2, the skin friction coefficient decreases with Mach number M for both small and large values of ξ .

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APPENDIX

Values of $q'_n(0)$ and $q'_{in}(0)$ (when $\sigma = 1, \gamma = 1.4$)

l	n	$T_w/T_s = 0.2$	0.2	0.2	0.2	0.2	2	2	2
		$m_1 = 0$	0.5	0	0.5	-	0	0.5	-
		$\beta = 0$	0	0.5	0.5	1	0.5	0.5	1
	0	-0.4757	-0.6865	-0.4823	-0.6099	-0.4789	-0.6678	-0.8126	-0.7371
	1	-0.1194	-0.0430	-0.1267	-0.0773	-0.9928	0.0227	0.0061	-0.0012
	2	-0.0156	0.0009	0.0309	0.0173	0.0169	0.0059	-0.0068	0.0050
	3	-0.0097	0.0005	-0.0088	-0.0051	-0.0038	0.0019	0.0013	0.0008
1	0	0.1338	-0.5714	-0.3201	-0.6052	-0.2391	-0.7378	-1.0356	1.6974
1	1	-0.5137	0.0083	-0.3364	-0.1223	-0.2756	0.1142	0.2296	0.1126
1	2	0.1142	-0.0187	0.1503	0.0749	0.0924	-0.0548	-0.0573	-0.0299
1	3	-0.0084	0.0056	-0.0543	0.0309	-0.0253	0.0144	0.0096	0.0050
2	0	0	0	0.0552	0.0523	0.0713	0.0354	0.0323	0.0422
2	1	0	0	-0.0602	-0.0445	-0.0611	-0.0259	-0.0180	-0.0237
2	2	0	0	0.0333	0.0214	0.0368	0.0096	0.0057	0.0103
2	3	0	0	-0.0129	-0.0076	-0.0197	-0.0025	-0.0013	-0.0043

Values of $P'_n(0)$ and $P'_{in}(0)$ (when $\sigma = 1, \gamma = 1.4$)

	0		0.3933	0.3447	0.6613	0.3134	0.2734	0.5061
	1		-0.1141	-0.0814	-0.1037	-0.0498	-0.0340	-0.0347
	2		0.0395	0.0252	0.0237	0.0093	0.0057	0.0027
	3		-0.0125	-0.0074	-0.0058	-0.0016	-0.0009	-0.0002
1	0		1.3556	1.1957	2.2354	1.0613	0.9337	1.6974
1	1		-0.6262	-0.4507	-0.5345	-0.2787	-0.1987	-0.1976
1	2		0.2798	0.1842	0.1471	0.0706	0.0436	0.0230
1	3		-0.1056	-0.0644	-0.0396	-0.0154	-0.0087	-0.0024
2	0		-0.0015	-0.0012	-0.0022	-0.0005	-0.0004	-0.0004
2	1		0.0012	0.0010	-0.0000	-0.0002	0.0002	-0.0002
2	2		-0.0007	-0.0005	0.0006	-0.0001	-0.0001	0.0002
2	3		0.0003	0.0002	-0.0005	-0.0000	0.0000	-0.0001

Séchage d'un milieu poreux contenant une faible teneur en eau

S. BEN NASRALLAH et G. ARNAUD

Laboratoire d'Etudes des Systèmes Thermiques et Energétiques (UA 1098),
40, Avenue du Recteur Pineau, 86022—Poitiers Cédex, France

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I. INTRODUCTION

LE SÉCHAGE est une opération rencontrée dans de nombreux secteurs industriels (industrie agroalimentaire, industrie du bâtiment, traitement du bois etc.). Une meilleure compréhension des phénomènes physiques intervenant dans cette opération peut permettre d'améliorer et d'optimiser les techniques du séchage. A ce titre, l'étude des transferts de chaleur et de masse lors du séchage des milieux poreux attire, depuis plusieurs décennies, l'attention de nombreux auteurs et fait l'objet de travaux aussi bien théoriques qu'expérimentaux. Citons les travaux de Whitaker [1-3] et de Marle [4] qui ont notamment pour objet la formulation mathématique du problème, de Bories *et al.* [5], de Moyne *et al.* [6] concernant la séchage à haute température, celui de Basilico *et al.* [7] pour le séchage du bois, de Haramaty [8], de Huang *et al.* [9, 10]. ... On retrouvera une bibliographie plus complète dans la Réf. [1].

Notre contribution consiste en l'étude théorique des trans-

ferts unidimensionnels de chaleur et de masse lors du séchage d'un milieu poreux contenant de l'eau en quantité faible (à l'état pendulaire). La matrice solide est inerte et indéformable. La phase gazeuse est constituée d'air et de vapeur d'eau. Le milieu poreux (Fig. 1) est adiabatique et imperméable sur une face, l'autre face étant perméable et en contact d'un écoulement d'air de température et d'humidité fixées.

Le problème est formulé en s'inspirant de la théorie de Whitaker [1]. La résolution numérique du modèle mathé-

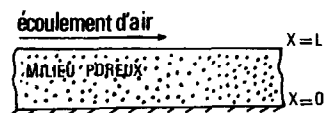


FIG. 1